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A PRODUCTION FUNCTION FOR TRAINED RECRUITS

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I. INTRODUCTION AND BACKGROUND

The Navy operates 3 recruit training bases at Great Lakes, Illinois, at San Diego, California, and at Orlando, Florida. In these training operations resources are used to turn raw recruits into trained personnel. The resources used to produce these trained personnel include labor, primarily in the form of enlisted instructors, and capital, largely composed of barracks, but also including classrooms and other buildings, as well as training equipment.

The purpose of this study is to determine the output capability, or capacity, of the Navy's initial entry training bases under current and alternative operating policies, as well as under various output requirements associated with alternative force levels. It does this by estimating: 1) the feasible output of trainees that can be obtained with the resources currently at the recruit training bases; 2) the surge capability of the bases if extra instructors are assigned to the recruit training commands (RTC's); and 3) the capacity of the recruit training establishment for any combination of facilities and instructors at the bases. This permits estimates of potential trainee output in the long-run, in which facilities as well as instructors may be varied. This last result will allow calculation of the mix of capital and labor for which the Navy should strive, given the relative prices

of buildings and instructors, to minimize the long-run costs of producing any desired number of trainees.

This paper will also examine the question of whether the Navy should have more or less than 3 training bases, and how output should be divided among these bases in the short-run. The basic framework of the study is an econometric estimation of a production function for trained recruits. A brief discussion of production functions is contained in the appendix. In the next section of this paper, a model of recruit training is presented, which is followed by the basic statistical results. The final section interprets these results.

II. THE MODEL

The analysis assumed that the number of recruits that a base can turn out in a year is given by:

$$T_i = f(K_i^t, L_i^t, R_i) \qquad (1)$$

where:

T = trained recruits

R = recruit input

K = replacement value of capital (facilities under control of the RTC in thousands of 1966 dollars)

L = labor (enlisted-instructor) man-years

The subscript $_i$ indicates that an individual base is referred to. The superscript ^{t} indicates that the instructors and facilities are used for training.

This is the production function for recruit training. The process of screening, weeding out unsuitable recruits, is intimately connected with the process of training. Conceptually at least it requires the use of resources to decide who to weed out.

$$S_i = (K_i^s, L_i^s, R_i, \overline{T}) \qquad (2)$$

where:

S_i = number of men screened out

$\overline{T}$ = $\Sigma_i T_i$

The superscript ^{s} indicates that the instructors and facilities are used for screening.

$\overline{T}$ is included in this equation as a measure of the demand by the Navy for trained recruits, since when the demand for trained recruits is high a smaller proportion of recruits is likely to be flunked out of boot camp.

It is difficult to divide the labor and capital, L and K, at a base into a component used for training and a component used for screening. In order to avoid this problem we make several assumptions:

$$K_i^s \ll L_i^t \quad \text{and} \quad I_i^s \ll I_i^t \qquad (3)$$

Therefore equation (2) can be re-written:

$$T_i = f(K_i, L_i, R_i) \qquad (4)$$

Also K_i^s and L_i^s are assumed to not be effective constraints on the screening process, that is:

$$K_i^s \simeq 0 \quad \text{and} \quad L_i^s \simeq 0 \qquad (5)$$

so that:

$$S_i = g(R_i, \overline{T}) \qquad (6)$$

This function describes the screening behavior of the Navy at different input-requirement levels. The behavioral relationship is assumed to be of the form:

$$\frac{S_i}{R_i} = A_i = g(\bar{T}) \tag{7}$$

where A_i is the attrition rate.

Thus, when $\bar{T}$ is known the attrition rate can be found. Knowing the number of trainees the Navy wants to turn out at a base, the number of recruits who must be sent to that base can be estimated. That is:

$$R_i = h(T_i) \tag{8}$$

and

$$T_i = f(K_i, L_i, h(T_i)) \tag{9}$$

or

$$T_i = G(K_i, L_i) \tag{10}$$

Equation (1) is not, strictly speaking, a production function because it incorporates the assumption that the Navy's policy with respect to attrition will continue to be what it has been in the past. Because of this the possibility of training more men by just taking in more recruits, without adding instructors or facilities, cannot be examined.

In line with the discussion above, equation (10) has the form:

$$T = MK^{\alpha}L^{\beta}$$

or

$$\log T = \log M + \alpha \log K + \beta \log L \tag{11}$$

Assuming unchanged screening behavior, this form allows estimates of the economies of scale $(\alpha+\beta)$ and of the elasticity of trained recruites with respect to both facilities and instructors.

Equation (7) was assumed to have the form:

$$A_i = N - \gamma \log \overline{T} + \epsilon d_o \tag{12}$$

N, γ, and ϵ are positive constants to be estimated and d_o is a dummy variable which takes the value 1 when the base being examined is Orlando and zero otherwise. The dummy variable is meant to capture the fact that, perhaps because of its newness (Orlando has only been open since 1968), Orlando has had much higher attrition rates than either Great Lakes or San Diego. The results in section III lend support to the functional form in equation (12).

A possible difficulty with the model is that it may not estimate a production function for trainees of constant quality, but rather estimates how the Navy behaves when manpower requirements are changed and that this behavior may incorporate fluctuations in the quality of trainees as well as fluctuations in the quantity of inputs.

There are 2 possible responses to this challenge. First a behavior function is of some interest by itself. But second, the criticism does not seem to be valid. The model is a production function for men of the quality the Navy produced at any level of manpower requirements. An empirical examination of final written tests given at San Diego between 1966 and 1971 indicates no correlation between test score and number of

trainees (correlation coefficient = .00459). Thus it appears men of the same quality are produced at all levels of output. The production function seems to be a production function for men of constant quality.

III. STATISTICAL ESTIMATE OF RESULTS

The values of the unknown parameters in the model described above can be estimated. Annual data for the years 1964-1969 for each of the bases was used in both equations. Since Orlando's first full year of operation was 1969 there were 13 annual observations in our sample. The data used in this study is displayed in Table I.

Standard statistical techniques were used to fit the data in the sample to our specified functional forms. These techniques have the desirable property of minimizing the sum of squared deviations of the estimated level of output from the actual level.

The estimated equations are:

$$\log T = 1.636 + \underset{(2.12)}{.244} \log K + \underset{(4.05)}{.724} \log L \qquad (13)$$

$$R^2 = .865$$

and

$$A = 130.209 - \underset{(4.00)}{25.117} \log \overline{T} + \underset{(2.56)}{2.946}\, d_o \qquad (14)$$

$$R^2 = .663$$

The numbers in parentheses are t-values.

TABLE I

BASIC DATA ON RECRUIT TRAINING*

BASE	YEAR	CAPITAL	ENLISTED INSTRUCTORS	RAW RECRUITS	ATTRITION Rate %	OUTPUT
S.D.	1964	16800	586	43782	6.81	40800
	1965	16255	574	52139	5.07	49496
	1966	17605	593	51006	2.87	49542
	1967	16464	594	46211	2.22	45185
	1968	15065	536	51979	2.85	50498
	1969	14770	549	45866	4.27	43908
	1970	15770	521	33610	5.03	31921
G.L.	1964	29844	640	45809	7.38	42428
	1965	31230	637	61650	4.10	59122
	1966	29741	618	65890	2.69	54118
	1967	39690	547	59235	2.75	57606
	1968	39212	573	68628	2.90	66638
	1969	39552	654	56620	4.17	54259
	1970	38950	632	43203	2.92	41943
ORL.	1969	7122	182	16517	6.18	15497
	1970	11808	210	17687	11.32	15635

*Capital is the replacement cost of barracks and other RTC facilities in thousands of 1966 dollars. Basic sources are NavFac P-164 and PERS C. *Enlisted instructors are in man-years.

These results are striking, especially equation (13). All the coefficients are sizeable and have the expected signs. The percentages of explained variation of the dependent variables (R^2) are as high as can be reasonably be hoped for in regressions run on data from the real world. The average residual from equation (13) is only 10% of the actual number of trainees.

IV. INTERPRETATION OF RESULTS

1. Returns to Scale

#Most interesting is the strong indication of constant returns to scale. The sum of the coefficients of capital and labor in equation (13), .97, is so close to 1.0 as to be undistinguishable from it.* The implication of this finding is that there is no strong reason to increase or decrease the number of recruit training bases which the Navy has in operation.

2. Present Capacity

The average quantities of instructors and facilities on hand at the recruit training centers in 1970 (the latest year for which data are available) were:

	Facilities	Instructors	Capacity Output
Orlando	$11.81 million	210	20,423
San Diego	15.77 million	521	42,322
Great Lakes	38.95 million	632	60,676
Total	$66.53 million	1,363	123,421

*Slight changes in the capital variable used in this equation which we tried had coefficients which summed to 1.0 almost exactly. We chose the form shown here because of its slightly higher R^2.

Thus, according to equation (13), there is a total capacity to train 123,421 recruits in a nine-week course. Actual output in 1970 was about 90,000 men. However, an 11-week course was used from January, 1970 to May, 1971. Even so, if the eleven-week trainees used 11/9 as many resources as nine-week trainees, the capacity existed to train 11,000 more men in 11-week courses or 13,000 more men in 9-week courses.* Stated another way, there were 258 more enlisted instructors than were necessary to turn out the number of men produced in the existing facilities. This is simply a reflection of the fact that the number of instructors has not been reduced in line with the recent reduction in recruit requirements from the Viet Nam levels. It is assumed that the same seasonal variations in the recruit load will occur in the future as they have in the past. The number of instructors required is based on historical experience where the annual average number of instructors reflects this seasonality. Since the Navy cannot instantaneously adjust the number of instructors to monthly or weekly changes in the recruit load, it assigns instructors to the RTC's based upon some average workload to accommodate seasonal fluctuations. The same is true of these numbers. That is, if 258 instructors were to be cut from the RTC's, the remaining instructors would have larger companies in the summer and smaller ones in the winter, as they have in the past.

Finally, note that the number of recruits produced does not include a small number of reservists who go through a two-week course. Since the number of instructors is actual, they trained a slightly larger fraction of recruits than these numbers imply. Assuming that the short-course reservists continue to cycle into the RTC's as they have in the past, the required number of instructors will reflect and accommodate this.

*Since May, 1971 the RTC's have returned to a nine-week course.

3. Surge Capacity Using More Instructors

According to equation (13) increases in the output of any training base in the short-run may be obtained by adding more instructors to the fixed facilities at the RTC's. Indeed any size increase in trained recruits may be gained in this fashion, but at increasing cost. Solving equation (13) for the logarithm of L shows that the number of instructors necessary to turn out the desired number of trainees, T, at a base with capital of value K_o is given by:

$$\log L = -2.26 + 1.38 \log T - .34 \log K_o \tag{15}$$

where K is the replacement cost of barracks and other RTC facilities measured in thousands of 1966 dollars. Thus, the number of instructors needed to accommodate a surge of any desired size can be estimated. Table II shows the number of instructors that would be necessary at each of the recruit training bases to produce different numbers of trained recruits given the 1970 capital stocks at the bases. It is possible to go further than this, however.

If the Navy is producing the same kind of trained recruits at many training bases with fixed (in the short-run) facilities it can minimize the cost of any total amount of output by equalizing the marginal products of variable inputs at all bases. For any desired total number of trainees, $\overline{T}$, in a year, and any existing facilities at the three training bases, K_1, K_2, and K_3, equations (13) and (14) together with the equal marginal product rule, make it possible to calculate:

1. the optimum output of each of the training bases;
2. the optimum distribution of instructors among bases; and
3. the implied number of raw recruits at each base.

Table III shows the optimum way of distributing various numbers of total trainees among the three bases, given the 1970 level of facilities at the bases. The last two lines of the table show the optimum distribution of 1969 output among the bases, given the 1969 facilities and compares this optimum with the actual distribution of trained recruits. Note that a shift of recruits and instructors from San Diego to Great

TABLE II

SURGE CAPACITY WITH 1970 FACILITIES*

TRAINED RECRUITS	*SAN DIEGO*	*GREAT LAKES*	*ORLANDO*
10000	71	52	78
20000	185	137	204
30000	324	239	357
40000	482	356	531
50000	656	484	723
60000	844	622	930
70000	1044	770	1150
80000	1255	926	1383
90000	1477	1089	1628
100000	1708	1260	1882

*Table entries are the number of instructors required to produce the specified number of trained recruits (row headings) at the specified base (column headings).

TABLE III

OPTIMAL DISTRIBUTION OF TRAINING WITH 1970 CAPITAL STOCK

Total trainees	Orlando trainees	Orlando instructors	Orlando recruits	S.D. trainees	S.D. instructors	S.D. recruits	G.L. trainees	G.L. instructors	G.L. recruits
50,000	9,683	75	11,410	12,506	97	14,241	27,811	215	31,671
60,000	11,620	96	13,379	15,007	124	16,711	33,373	277	37,163
70,000	13,557	119	15,312	17,508	154	19,138	38,935	343	42,560
80,000	15,494	143	17,216	20,009	185	21,529	44,497	412	47,878
90,000	17,430	169	19,096	22,510	218	23,890	50,060	485	53,128
100,000	19,367	195	20,954	25,011	252	26,225	55,622	560	58,320
110,000	21,304	223	22,793	27,513	287	28,536	61,184	639	63,460
120,000	23,240	251	24,615	30,014	324	30,827	66,746	721	68,554
130,000	25,177	280	26,422	32,514	362	33,099	72,308	805	73,607
140,000	27,114	311	28,215	35,016	401	35,354	77,870	892	78,622
150,000	29,050	342	29,995	37,517	441	37,594	83,433	981	83,603
Optimal mix for 1969 production and 1969 capital stock									
113,663	15,242	158	16,245	29,043	301	30,013	69,377	719	71,693
Actual mix for 1969 production									
113,663	15,497	182	16,517	43,908	549	45,866	54,258	654	56,620

Lakes would have permitted the Navy to reduce the number of instructors by 207 with no change in trained recruit output. Since the billet cost of an enlisted instructor is about $18,000, this implies a possible saving of approximately $3.7 million. In addition to being bigger than the RTC at San Diego, the RTC at Great Lakes is also newer. As of 1969, 78% of the barracks area at Great Lakes was in permanent buildings constructed since 1958. Only 29% of the barracks at San Diego were permanent and none of them were built after 1956. It seems that there is a strong case for shifting recruits and instructors from San Diego to Great Lakes.*

Table IV displays the optimal distribution of recruits among the three bases in FY '73. The Navy plans to train 100,000 men in FY '73 and by then an entire new section will be open at Orlando, costing $12.9 million, just about doubling the recruit training facilities at the base. Additions worth $2.339 million are being built at Great Lakes. No change is planned at San Diego.

Equalization of the marginal product of instructors at two bases requires that:

$$\frac{L_1}{L_2} = \left(\frac{K_2}{K_1} \right)^{\alpha/\beta-1} \quad **$$

where L_1 and L_2 are the number of instructors at the two bases. From this it follows that the relationship between the instructor-to-facilities ratios at the two bases should be:

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*Although any changes in transportation costs that may follow from this result were not explicitly analyzed, it appears that the net change might actually reduce transportation costs.

**α and β are as defined in equation (11).

TABLE IV

OPTIMAL DISTRIBUTION OF RECRUITS AND INSTRUCTORS WITH EXPECTED 1973 CAPITAL STOCKS

Base	Expected* Facilities	Trained Output	Recruit Input	Instructors
San Diego	$15,770	21,206	22,235	201
Orlando	22,900	29,489	30,919	279
Great Lakes	40,961	49,305	51,697	467
Total	$79,631	100,000	104,851	947

*replacement cost in thousands of 1966 dollars.

$$\frac{L_1/K_1}{L_2/K_2} = \left(\frac{K_2}{K_1}\right)^{(\alpha+\beta-1)/(\beta-1)}$$

$$= 1 \quad \text{if} \quad \alpha + \beta = 1$$

Thus, if there are constant returns to scale ($\alpha+\beta=1$), which there appear to be in recruit training, a good short-run rule is to equalize the labor-to-capital ratio among recruit training bases. In fact, the results in tables III and IV are quite close to what this simple rule would suggest. There is not exact correspondence because we estimated $\alpha + \beta = .97$ rather than 1.0.

4. Long Run Capacity

Equation (13) implies that the number of trainees turned out at a base can be produced with any one of an infinite number of combinations of facilities and instructors. Table V displays, for base outputs between 10,000 and 100,000, some of the feasible combinations of facilities and instructors. For instance, 30,000 trained recruits can be turned out with either 299 instructors and $20 million worth of facilities, 213 instructors and $55 million worth of facilities, or a whole range of other combinations.

These combinations of facilities and instructors also can be graphically displayed. For example, figure 1 demonstrates the smooth relationship among combinations of facilities and instructors which can turn out 50,000 trainees at a base in a year.

The question then arises: what particular combination of facility and instructors should the Navy use to produce the output desired at each base? The mix of inputs which produces the desired output at

TABLE V

ALTERNATIVE WAYS OF TRAINING DIFFERENT NUMBERS OF MEN AT A BASE*

CAPITAL thousands of 1966 dollars \ OUTPUT OF TRAINED RECRUITS	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
5000	105	272	477	709	965	1242	1536	1847	2173	2514
10000	83	216	378	562	765	983	1217	1463	1721	1991
15000	72	188	329	490	667	858	1061	1276	1502	1737
20000	66	171	299	445	605	779	963	1159	1363	1577
25000	61	159	277	413	562	722	894	1075	1265	1463
30000	57	149	261	388	528	679	841	1011	1189	1375
35000	54	142	248	369	502	645	798	960	1129	1306
40000	52	135	237	352	479	617	763	918	1080	1249
45000	50	130	228	339	461	593	733	882	1038	1200
50000	48	126	220	327	445	572	708	851	1001	1158
55000	47	122	213	317	431	554	685	824	970	1122
60000	45	118	207	307	418	538	666	800	942	1089
65000	44	115	201	299	407	524	648	779	917	1060
70000	43	112	196	292	397	511	632	760	894	1034
75000	42	110	192	285	388	499	618	743	874	1011
80000	41	107	188	279	380	488	604	727	855	989
85000	40	105	184	273	372	479	592	712	838	969
90000	40	103	180	268	365	469	581	698	822	950
95000	39	101	177	263	358	461	570	686	807	933
100000	38	99	174	259	352	453	561	674	793	917
105000	38	98	171	255	347	446	551	663	780	902
110000	37	96	169	251	341	439	543	653	768	888
115000	36	95	166	247	336	432	535	643	757	875
120000	36	93	164	243	331	426	527	634	746	863
125000	35	92	161	240	327	420	520	625	736	851

*Entries in the table are the number of instructors necessary to train the specified number of men (column headings) with the specified amount of facilities (row headings).

ALTERNATIVE INPUT COMBINATIONS TO 50,000 MEN

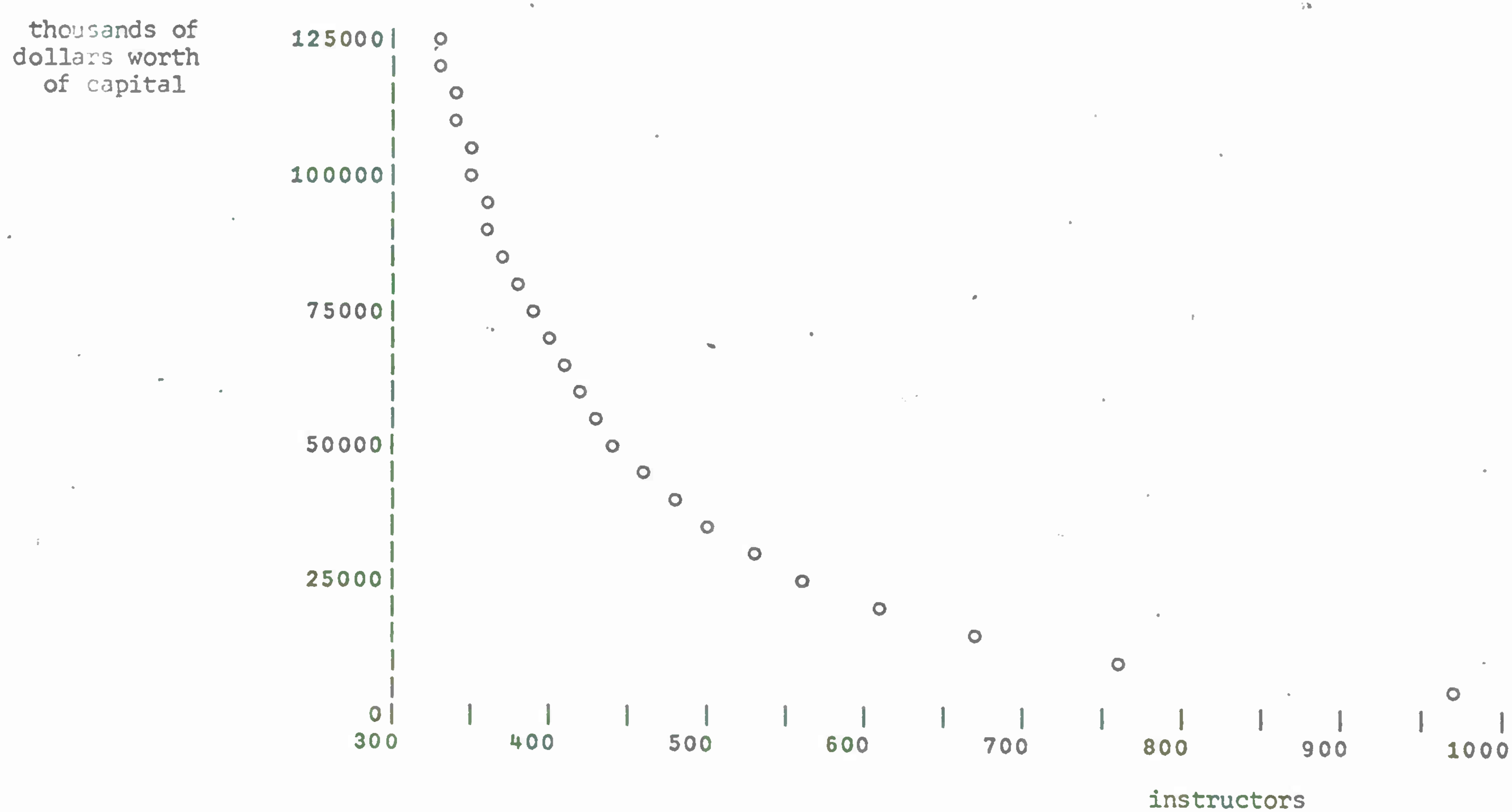

FIG. 1

the lowest cost is that for which the ratio of the marginal products of the inputs equals the ratio of the prices of the inputs. That is,

$$\frac{MP_K}{MP_L} = \frac{P_K}{P_L} \tag{16}$$

In a single year, equation (6) tells us that:

$$MP_K^o = \alpha\frac{T}{K} = .244\frac{T}{K} \tag{17}$$

and

$$MP_L^o = \beta\frac{T}{L} = .724\frac{T}{L} \tag{18}$$

where MP_F^o is the marginal product of facilities in one year and MP_I^o is the marginal product of instructors in one year. In the case of instructors, the marginal product in the current year is bought for the current annual pay, P_I^o. Thus $\frac{P_I}{MP_I} = \frac{P_I^o}{MP_I^o}$. However, capital is long lived and continues to have a marginal product long after it is purchased. The marginal product of facilities in present value terms is:

$$\begin{aligned} MP_K &= \alpha\frac{T}{K} + \frac{\alpha\frac{T}{K}}{1+r} + \frac{\alpha\frac{T}{K}}{(1+r)^2} + \ldots \\ &= \sum_{i=0}^{\infty} \frac{\alpha\frac{T}{K}}{(1-r)^i} = \frac{\alpha\frac{T}{K}}{r} \end{aligned} \tag{19}$$

where r is the fraction by which the Navy prefers a trained recruit today to a trained recruit next year. It is the Navy's discount rate.

Also the cost of facilities is not simply their price. Facilities depreciate at a rate δ and require maintenance at a rate m. The full price of maintaining facilities in present value terms is, then:

$$P_K = P_K^o + P_K^o(\delta+m) + \frac{P_K^o(\delta+m)}{(1+r)} + \frac{P_K^o(\delta+m)}{(1+r)^2} + \ldots$$

$$= P_K^o\left[1 + \sum_{i=0}^{\infty} \frac{\delta+m}{(1+r)^i}\right] = P_K^o\left[1 + \frac{\delta+m}{r}\right] = P_K^o\left[\frac{r+\delta+m}{r}\right] \qquad (20)$$

The depreciation and maintenance expenditures are discounted because the value of what the money spent replacing depreciated facilities and maintaining it in the future could buy is less than what the same sum (corrected for inflation) could buy today. Note that depreciation could have been subtracted in the marginal product equations rather than being added in the price equations. The result is the same. Writing a constant (undiscounted) stream of productivity requires that depletions in the stock of capital due to depreciation be replaced--which entails expenditures at the rate δ.

Equations (16) to (20) imply that:

$$\frac{\frac{\alpha_K^T}{r}}{\beta_L^T} = \frac{P_K^o\left[\frac{r+\delta+m}{r}\right]}{P_L} \quad \text{or}$$

$$\frac{.244L}{.724K} = \frac{P_K^o\left[r+\delta+m\right]}{P_L} \qquad (21)$$

The optimum instructor to plant ratio, $\left(\frac{L}{K}\right)$, is

$$\left(\frac{L}{K}\right)^* = \frac{.724}{.244}\;\frac{P_K^o\left[r+\delta+m\right]}{P_L} = 2.97\;\frac{P_K^o\left[r+\delta+m\right]}{P_L} \qquad (22)$$

The Naval Facilities Engineering Command has estimated $\delta = .02$ for barracks. NAVFAC also has provided data which allows the estimation of $m = .03$. There has been considerable controversy over what the proper discount rate is for the Navy's use. There has been a tendency to use $r = .1$, although there is a question about whether an entity with an apparently infinite life expectancy and no ability to invest funds for monetary return should discount the future. We therefore calculated $\left(\frac{L}{K}\right)$ for $r = .1$ and $r = 0$.

If $r = 0$ is appropriate, equation (13) implies:

$$\widehat{\left(\frac{L}{K}\right)} = 2.97\,\frac{1000(.05)}{1800} = .0082 \qquad (23)$$

If $r = .1$:

$$\widehat{\left(\frac{L}{K}\right)} = 2.97\,\frac{1000(.15)}{1800} = .025 \qquad (24)$$

In fact, in 1970, $\frac{L}{K} = .033$ at San Diego, .016 at Great Lakes, and .018 at Orlando. For the recruit training establishment as a whole, $\frac{L}{K} = .02$. Table VI shows, for each of the three bases producing the same output of trained recruits that were turned out in 1969, the optimum mix of capital and labor inputs, both under the assumption that $r = 0$ and that $r = .1$. The actual input mixes are also shown and potential savings from mix changes are calculated. It is important to note that this table does not indicate the optimum way to operate the bases which now exist at San Diego, Great Lakes, and Orlando since

the bases cannot freely vary both facilities and instructors. It does indicate how to operate if the decision is made to aim for a long-run flow of output of about 110,000 per year distributed among three bases as the 1969 output was.

Table VI suggests that, regardless of the discount rate, the Navy appears to be using too many enlisted instructors, although the magnitude of this misallocation depends considerably on the discount rate. Column (2) shows that Navy recruit training should use more facilities if there is no rate of discount. Column (4) largely illustrates the previously noted fact that the plant at San Diego was over-utilized in 1969 while that at Great Lakes was underutilized. Notice that the long-run optimum total number of teachers in column (4), 1305, is 127 more than the short-run optimum shown in table III. This is because the 1969 total facilities value is above the optimum total amount implied by $r = .1$.

Thus, it is difficult to determine whether Navy training was too capital intensive or not because of uncertainty about the appropriate discount rate. We can, however, say that there is money to be saved by equalizing the facilities-to-instructor ratios at the three bases.

V. SUMMARY AND CONCLUSIONS

1. *Navy training exhibits constant returns to scale. This implies that there is no reason to increase or decrease the number of recruit training bases.

TABLE VI

THE ESTIMATED LONG RUN OPTIMAL MIX OF CAPITAL AND LABOR BY RECRUIT TRAINING BASE

	r=0		r=.1	
	(1) Actual	(2) Optimal	(3) Actual	(4) Optimal
Total output	113,663	113,663	113,663	113,663
San Diego output	43,908	43,908	43,908	43,908
instructors	549	381	549	505
facilities	14,680	46,470	14,680	20,180
cost	$10,616,000	$9,181,150	$12,084,000	$12,117,000
Great Lakes output	54,258	54,258	54,258	54,258
instructors	654	474	654	628
facilities	39,552	57,830	39,552	25,110
cost	$13,749,600	$11,423,500	$17,704,800	$15,070,500
Orlando output	15,497	15,497	15,497	15,497
instructors	182	130	182	172
facilities	7,122	15,846	7,122	6,881
cost	$3,632,100	$3,132,300	$4,344,300	$4,128,150
Total cost	$27,997,100	$23,736,950	$34,133,100	$31,315,650

2. In 1970 there was excess capacity. That is, there were more enlisted instructors than were necessary to turn out the number of men produced in the existing plant.

3. Tables II and III provide estimates of the surge capability of the training bases as well as a guide for responding to a surge in the demand for trained recruits.

4. Table IV is a guide to operation in FY '73. It says that, due to facilities expansion, fewer instructors will be needed by the RTC's.

5. The analysis suggests that the base at San Diego is used too intensively. Trainees and instructors should be shifted from there to Great Lakes. This might yield an annual saving of $3 million, or more.

6. Table V and figure 1 display alternative means of combining instructors and facilities to produce the desired number of trainees at a base.

7. The technique for calculating optimum capital-to-labor ratios at the three bases was outlined and such ratios were calculated under various assumptions. This ratio should be equalized among the three bases and the relative outputs of the bases adjusted accordingly. It does not matter where men are trained, because of constant returns to scale. Equalization of the capital-to-labor ratio among bases assures that men will be trained at minimum cost. Table VI illustrates this.

APPENDIX

PRODUCTION FUNCTIONS AND CAPACITY

The capacity of a plant, or a training base, is not uniquely determined simply by its physical size. A training base with a certain amount of equipment can produce more trainees if the plant is used more intensively, that is, if more instructors are added. Likewise a given number of instructors can produce a larger output if they have more capital at their disposal. It follows that a particular quantity of trained recruits can be produced using different combinations of facilities and instructors. Since, in most production processes, it takes longer to change the facilities used in production than it does to change the number of instructors, short-run fluctuations in output are generally accomplished by changing the size of the work force. Longer-run fluctuations in output will generally be accompanied by changes in training base size as well.

A convenient tool that mathematically combines the notion of capacity with the technological relationships underlying the production of a particular commodity is the production function. A production function associates a level of output, T, with each combination of inputs, F and I, where F may be facilities, I instructors, and R new recruit accessions:

$$T = f(F,I,R)$$

In general production functions have the characteristics that 1) an increase in the level of any input should produce an increase in the

level of output and 2) subsequent increases in the level of any one input, holding all other inputs constant, should produce smaller and smaller increases in the level of output. That is:

$$\frac{\partial T}{\partial F} > 0 \; ; \; \frac{\partial^2 T}{\partial F^2} < 0$$

$$\frac{\partial T}{\partial I} > 0 \; ; \; \frac{\partial^2 T}{\partial I^2} < 0$$

$$\frac{\partial T}{\partial R} > 0 \; ; \; \frac{\partial^2 T}{\partial R^2} < 0$$

The form of the production function used in this study allows estimates of the percent by which output can be expanded by a one percent increase in the number of instructors or a one percent increase in the amount of capital at a base. These amounts of expansion are called the elasticities of output with respect to the varying input. Returns to scale in a production process can also be estimated. If there are increasing returns to scale (output more than doubles when the inputs are doubled) there is a case to be made for concentrating production at one location. If there are decreasing returns to scale (output less than doubles) many small operations are indicated. If there are constant returns to scale (output exactly doubles) the number of plants or bases in operation is essentially a matter of indifference.

There are a number of accepted forms of the production function which might be applied to training bases. This paper concentrates primarily on the form:

$$X = MF^{\alpha} I^{\beta} R^{\delta} \tag{A-1}$$

In this formulation the elasticity of output with respect to an input is the exponent of that input. The degree of homogeneity, which measures the returns to scale, is the sum of the exponents $(\alpha+\beta+\delta)$ and is not allowed to change with changes in input levels. The elasticity of substitution between factors is a measure of how well two factors substitute for each other at a particular level of all inputs. A value of zero implies that the two factors in question cannot replace each other at all. A value of infinity indicates perfect substitutability. The form used imposes a value of one on the elasticity of substitution, and thus does not allow independent estimation of this parameter.

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